Investigation of the integral formula $\int \frac{x^{m-1}dx}{(1+x^k)^n}$ in the case in which one set $x = \infty$ after the integration *

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1. It is certainly already well-known that the integral of the formula in the case $n = 1$ contains partially logarithms and partially circular arcs and the logarithmic parts constitute this progression

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$$-\frac{2}{k}\ \cos\frac{m\pi}{k}\ \log\sqrt{1-2\cos\frac{\pi}{k}+xx}$$

$$-\frac{2}{k}\cos\frac{3m\pi}{k}\ \log\sqrt{1-2\cos\frac{\pi}{k}+xx}$$

$$-\frac{2}{k}\cos\frac{5m\pi}{k}\ \log\sqrt{1-2\cos\frac{\pi}{k}+xx}$$

$$-\frac{2}{k}\cos\frac{7m\pi}{k}\ \log\sqrt{1-2\cos\frac{\pi}{k}+xx}$$

$$\vdots$$

$$-\frac{2}{k}\ \cos\frac{im\pi}{k}\ \log\sqrt{1-2\cos\frac{\pi}{k}+xx},$$

where i denotes an odd number not greater than k. Hence, if k was an even number, it will be $i=k-1$; and if k was an odd number, this progression must be continued to $i=k$, but its coefficients must be taken half as small or instead of $-\frac{2}{k}$ one has to write $-\frac{1}{k}$, the reason for which irregularity was explained in *Calculi Integralis*.

2. Since these parts vanish for $x=0$, let us immediately put $x=\infty$, and since in general

$$\sqrt{1-2x\cos\omega+xx}=x-\cos\omega,$$

it will be

$$\log\sqrt{1-2x\cos\omega+xx}=\log(x-\cos\omega)=\log x-\frac{\cos\omega}{x}=\log x$$

because of $\frac{\cos\omega}{x}=0$; therefore, all these logarithms are reduced to the same form $\log x$, which is to be multiplied by this series

$$-\frac{2}{k}\cos\frac{m\pi}{k}-\frac{2}{k}\cos\frac{3m\pi}{k}-\frac{2}{k}\cos\frac{5m\pi}{k}-\cdots-\frac{2}{k}\cos\frac{im\pi}{k},$$

where, as we said, i denotes the largest odd number not greater than k, just with that restriction that, if k was an odd number and hence $i=k$, the last term must reduced by its half. Therefore, if we want to investigate the sum of this progression, two cases must be considered, the one, in which k is an even number and $i = k - 1$, the other, in which k is odd and $i = k$.

EXPANSION OF THE FIRST CASE IN WHICH k IS AN EVEN NUMBER AND $i = k - 1$

3. Therefore, in this case having put $x = \infty$ the formula $-\frac{2}{k}\log x$ is multiplied by this series

$$\cos\frac{m\pi}{k} + \cos\frac{3m\pi}{k} + \cos\frac{5m\pi}{k} + \cos\frac{7m\pi}{k} + \cdots + \cos\frac{(k-1)m\pi}{k},$$

whose sum we want to set $= S$. Let us multiply this series by $\sin\frac{m\pi}{k}$, and since in general

$$\sin\frac{m\pi}{k}\cos\frac{im\pi}{k} = \frac{1}{2}\sin\frac{(i+1)m\pi}{k} - \frac{1}{2}\sin\frac{(i-1)m\pi}{k},$$

after this reduction we will have

$$\begin{aligned} &S\sin\frac{m\pi}{k} \\ &= \frac{1}{2}\sin\frac{2m\pi}{k} + \frac{1}{2}\sin\frac{4m\pi}{k} + \frac{1}{2}\sin\frac{6m\pi}{k} + \cdots + \frac{1}{2}\sin\frac{(k-2)m\pi}{k} + \frac{1}{2}\sin\frac{km\pi}{k} \\ &-\frac{1}{2}\sin\frac{2m\pi}{k} - \frac{1}{2}\sin\frac{4m\pi}{k} - \frac{1}{2}\sin\frac{6m\pi}{k} - \cdots + \frac{1}{2}\sin\frac{(k-2)m\pi}{k}, \end{aligned}$$

where obviously all terms except for the last cancel so that

$$S\sin\frac{m\pi}{k} = \frac{1}{2}\sin m\pi.$$

But now, since our coefficients m and k are supposed to be integers, it will obviously be $\sin m\pi = 0$ and hence $S = 0$, unless it also was $\sin\frac{m\pi}{k} = 0$, which case can not occur, since in the integration of the propounded formula $\frac{x^{m-1}dx}{(1+x^k)^n}$ one must always assume $m < k$. Therefore, this way we have shown that in this case, in which one sets $x = \infty$ after the integration, all logarithmic parts of the integral cancel.

EXPANSION OF THE OTHER CASE IN WHICH k IS AN ODD NUMBER AND $i = k$

4. Therefore, in this case, having taken $x = \infty$, the formula $\log x$ is multiplied by this series

$$-\frac{2}{k}\cos\frac{m\pi}{k} - \frac{2}{k}\cos\frac{3m\pi}{k} - \frac{2}{k}\cos\frac{5m\pi}{k} - \cdots - \frac{2}{k}\cos\frac{km\pi}{k},$$

where the penultimate term is $\frac{2}{k}\cos\frac{(k-2)\pi m}{k}$, but for the last term it will be $\cos m\pi = \pm 1$ while the upper sign holds, if m is an even number, but the lower sign, if m is odd; hence having removed the last term, for the remaining terms let us set

$$\cos\frac{m\pi}{k} + \cos\frac{3m\pi}{k} + \cos\frac{5m\pi}{k} + \cdots + \cos\frac{(k-2)m\pi}{k} = S,$$

so that the multiplicator of the logarithm of x is

$$-\frac{2S}{k} - \frac{1}{k}\cos m\pi.$$

Hence proceeding as before it will be

$$\begin{aligned} & S\sin\frac{m\pi}{k} \\ = & \frac{1}{2}\sin\frac{2m\pi}{k} + \frac{1}{2}\sin\frac{4m\pi}{k} + \frac{1}{2}\sin\frac{6m\pi}{k} + \cdots + \frac{1}{2}\sin\frac{(k-2)m\pi}{k} + \frac{1}{2}\sin\frac{km\pi}{k} \\ & -\frac{1}{2}\sin\frac{2m\pi}{k} - \frac{1}{2}\sin\frac{4m\pi}{k} - \frac{1}{2}\sin\frac{6m\pi}{k} - \cdots + \frac{1}{2}\sin\frac{(k-2)m\pi}{k}, \end{aligned}$$

where again all terms except for the last cancel each other, so that hence it results

$$S\sin\frac{m\pi}{k} = \frac{1}{2}\sin\frac{(k-1)m\pi}{k} = \frac{1}{2}\sin\left(m\pi - \frac{m\pi}{k}\right);$$

but on the other hand

$$\sin\left(m\pi - \frac{m\pi}{k}\right) = \sin m\pi\cos\frac{m\pi}{k} - \cos m\pi\sin\frac{m\pi}{k},$$

where it should be noted that $\sin m\pi = 0$ because of the integer number m; therefore, we will have

$$S\sin\frac{m\pi}{k} = -\frac{1}{2}\cos m\pi \sin\frac{m\pi}{k} \quad \text{or} \quad S = -\frac{1}{2}\cos m\pi,$$

as a logical consequence the multiplicator of $\log x$ will be

$$= \frac{1}{k}\cos m\pi - \frac{1}{k}\cos m\pi = 0$$

and so it is manifest, no matter whether k is an even or odd number, that all logarithmic terms in our integral cancel each other, if we set $x = \infty$ after the integration, as we always assume here.

5. Now let us consider also the parts depending on the circle the integral of our formula is composed of. But these parts are seen to constitute the following progression:

$$\begin{gathered}
\frac{2}{k}\sin\frac{m\pi}{k}\arctan\frac{x\sin\frac{\pi}{k}}{1-x\cos\frac{\pi}{k}} + \frac{2}{k}\sin\frac{3m\pi}{k}\arctan\frac{x\sin\frac{3\pi}{k}}{1-x\cos\frac{3\pi}{k}} \\
+\frac{2}{k}\sin\frac{5m\pi}{k}\arctan\frac{x\sin\frac{5\pi}{k}}{1-x\cos\frac{5\pi}{k}} + \frac{2}{k}\sin\frac{7m\pi}{k}\arctan\frac{x\sin\frac{7\pi}{k}}{1-x\cos\frac{7\pi}{k}} \\
+\cdots+\frac{2}{k}\sin\frac{im\pi}{k}\arctan\frac{x\sin\frac{i\pi}{k}}{1-x\cos\frac{i\pi}{k}},
\end{gathered}$$

where in our last term either $i = k-1$ or $i = k$; the first of course holds, if i is an even number, the second, if an odd number.

6. But since all these terms vanish for $x = 0$, let us for our purposes set $x = \infty$. Therefore, in general it will be

$$\arctan\frac{x\sin\frac{i\pi}{k}}{1-x\cos\frac{i\pi}{k}} = \arctan\left(-\tan\frac{i\pi}{k}\right).$$

But on the other hand

$$-\tan\frac{i\pi}{k} = +\tan\frac{(k-i)\pi}{k},$$

from which this arc is $= \frac{(k-i)\pi}{k}$. Therefore, hence successively writing the numbers 1, 3, 5, 7 etc. for i these parts of our integral in question will be

$$\frac{2(k-1)\pi}{kk}\sin\frac{m\pi}{k}+\frac{2(k-3)\pi}{kk}\sin\frac{3m\pi}{k}+\frac{2(k-5)\pi}{kk}\sin\frac{5m\pi}{k}+\frac{2(k-7)\pi}{kk}\sin\frac{7m\pi}{k}$$
$$+\frac{2(k-9)\pi}{kk}\sin\frac{9m\pi}{k}+\cdots+\frac{2(k-i)\pi}{kk}\sin\frac{im\pi}{k},$$
where in the case, in which k is an even number, one has to proceed to $i=k-1$, and if k is an odd number, to $i=k$.

7. For the sake of brevity let us set

$$(k-1)\sin\frac{m\pi}{k}+(k-3)\sin\frac{3m\pi}{k}+(k-5)\sin\frac{5m\pi}{k}+\cdots+(k-i)\sin\frac{im\pi}{k}=S,$$

so that the integral in question is $\frac{2\pi S}{kk}$, since the logarithmic parts cancel each other. Now let us multiply both sides by $2\sin\frac{m\pi}{k}$, and since in general

$$2\sin\frac{m\pi}{k}\sin\frac{im\pi}{k}=\cos\frac{(i-1)m\pi}{k}-\cos\frac{(i+1)m\pi}{k},$$

after the substitution it will be

$$\begin{aligned}2S\sin\frac{m\pi}{k}&=(k-1)\cos\frac{0m\pi}{k}\\&+(k-3)\cos\frac{2m\pi}{k}+(k-5)\cos\frac{4m\pi}{k}+\cdots+(k-i)\cos\frac{(i-1)m\pi}{k}\\&-(k-1)\cos\frac{2m\pi}{k}-(k-3)\cos\frac{4m\pi}{k}-\cdots-(k-i+2)\cos\frac{(i-1)m\pi}{k}-(k-i)\cos\frac{(i+1)m\pi}{k}\end{aligned}$$

which series is manifestly contracted to the following one

$$2S\sin\frac{m\pi}{k}=k-1-2\cos\frac{2m\pi}{k}-2\cos\frac{4m\pi}{k}-2\cos\frac{6m\pi}{k}-\cdots-2\cos\frac{(i-1)m\pi}{k}$$
$$-(k-i)\cos\frac{(i+1)m\pi}{k},$$

where, having removed the first and last term, the intermediate terms constitute a regular series, for the investigation of whose value we want to put

$$T = \cos\frac{2m\pi}{k} + \cos\frac{4m\pi}{k} + \cos\frac{6m\pi}{k} + \cdots + \cos\frac{(i-1)m\pi}{k},$$

so that

$$2S\sin\frac{m\pi}{k} = k - 1 - 2T - (k-i)\cos\frac{(i+1)m\pi}{k}.$$

But here it is again convenient to consider two cases, depending on whether k was even or odd.

EXPANSION OF THE FIRST CASE IN WHICH k IS AN EVEN NUMBER AND $i = k-1$

8. Therefore, in this case we will have

$$T = \cos\frac{2m\pi}{k} + \cos\frac{4m\pi}{k} + \cos\frac{6m\pi}{k} + \cdots + \cos\frac{(k-2)m\pi}{k}.$$

Let us multiply by $2\sin\frac{m\pi}{k}$ and by the reductions indicated above we will have

$$\begin{aligned} 2T\sin\frac{m\pi}{k} &+ \sin\frac{3m\pi}{k} + \sin\frac{5m\pi}{k} + \cdots + \sin\frac{(k-3)m\pi}{k} + \sin\frac{(k-1)m\pi}{k} \\ -\sin\frac{m\pi}{k} &- \sin\frac{3m\pi}{k} - \sin\frac{5m\pi}{k} - \cdots - \sin\frac{(k-3)m\pi}{k} \end{aligned}$$

therefore, having cancelled the terms destroying each other, it will be

$$2T\sin\frac{m\pi}{k} = -\sin\frac{m\pi}{k} + \sin\frac{(k-1)m\pi}{k}.$$

But on the other hand

$$\sin\frac{(k-1)m\pi}{k} = \sin\left(m\pi - \frac{m\pi}{k}\right) = \sin m\pi\cos\frac{m\pi}{k} - \cos m\pi\sin\frac{m\pi}{k},$$

where $\sin m\pi = 0$, whence it will be

$$2T = -1 - \cos m\pi.$$

9. Having found the value for T one concludes that it will be

$$2S\sin\frac{m\pi}{k}=k \quad \text{and hence} \quad S=\frac{k}{2\sin\frac{m\pi}{k}}.$$

Finally, the value of our integral formulas we are after will be $\frac{2\pi S}{kk}$ and now it is manifest that the integral of our formula in the case in which S is an even number, will be $\frac{\pi}{k\sin\frac{m\pi}{k}}$, if one sets $x=\infty$ after the integration, of course.

EXPANSION OF THE OTHER CASE IN WHICH k IS AN ODD NUMBER AND $i=k$

10. Therefore, in this case

$$T=\cos\frac{2m\pi}{k}+\cos\frac{4m\pi}{k}+\cos\frac{6m\pi}{k}+\cdots+\cos\frac{(k-1)m\pi}{k},$$

which series multiplied by $2\sin\frac{m\pi}{k}$, as before, will produce

$$\begin{aligned}2T\sin\frac{m\pi}{k} &+ \sin\frac{3m\pi}{k}+\sin\frac{5m\pi}{k}+\cdots+\ \sin\frac{(k-2)m\pi}{k}+\sin\frac{(km\pi}{k}\\ -\sin\frac{m\pi}{k} &- \sin\frac{3m\pi}{k}-\sin\frac{5m\pi}{k}-\cdots-\sin\frac{(k-2)m\pi}{k},\end{aligned}$$

whence, having deleted the terms cancelling each other, one will find

$$2T\sin\frac{m\pi}{k}=-\sin\frac{m\pi}{k}+\sin m\pi$$

and hence

$$2T=-1+\frac{\sin m\pi}{\sin\frac{m\pi}{k}}=-1$$

because of $\sin m\pi=0$, and hence further

$$2S\sin\frac{m\pi}{k}=k;$$

hence, since the value in question of the integral is $\frac{2\pi S}{kk}$, also in this case our integral will be $=\frac{\pi}{k\sin\frac{m\pi}{k}}$, precisely as in the preceding case. Therefore, hence we deduce the following

THEOREM

11. *If this differential formula*

$$\frac{x^{m-1}dx}{1+x^k}$$

is integrated in such a way that, having put $x=0$, the integral vanishes, but then one sets $x=\infty$, the value resulting from this will always be

$$\frac{\pi}{k\sin\frac{m\pi}{k}},$$

no matter whether k is an even or and odd number.

The proof of this theorem is obvious from the preceding.

12. In the expansion of this formula we assumed that $m<k$, since otherwise the logarithmic terms would not have cancelled each other; but not even this restriction is necessary any longer. For, in the case $m=k$ the integral of the formula $\frac{x^{m-1}dx}{1+x^k}$ is $\frac{1}{k}\log\left(1+x^k\right)$, which for $x=\infty$ also is $x=\infty$; but the theorem indicates that our integral is $\frac{\pi}{k\sin\pi}=\infty$. Therefore, as long as m was not greater than k, our formula is always true.

13. Yes, it is not even necessary that the exponents m and k are integer numbers, as long as it was $m>k$; for, if it was $m=\frac{\mu}{\lambda}$ and $k=\frac{\varkappa}{\lambda}$, the value will be $\frac{\lambda\pi}{\varkappa\sin\frac{\mu\pi}{\varkappa}}$, the truth of which is shown this way. Since in that case one has to integrate this formula

$$\int\frac{x^{\frac{\mu}{\lambda}}}{1+x^{\frac{\varkappa}{\lambda}}}\cdot\frac{dx}{x},$$

set $x=y^{\lambda}$; it will be $\frac{dx}{x}=\frac{\lambda dy}{y}$ and the formula will become

$$\int\frac{y^{\mu}}{1+y^{\varkappa}}\cdot\frac{\lambda dy}{y}=\lambda\int\frac{y^{\mu-1}dy}{1+y^{\varkappa}},$$

whose value will obviously be $\frac{\lambda\pi}{\varkappa\sin\frac{\mu\pi}{\varkappa}}$.

Another Proof of this Theorem

14. Let P denote the value of the integral $\int \frac{x^m}{1+x^k} \cdot \frac{dx}{x}$ from the limit $x = 0$ to $x = 1$, but Q the value of the same integral from $x = 1$ to $x = \infty$, so that $P + Q$ yields the value contained in the theorem. Now, to find the value Q set $x = \frac{1}{y}$, whence $\frac{dx}{x} = -\frac{dy}{y}$, and it will be

$$Q = \int \frac{y^{-m}}{1+y^{-k}} \cdot \frac{-dy}{y} = -\int \frac{y^{k-m}}{1+y^k} \cdot \frac{dy}{y}$$

from $y = 1$ to $y = 0$. Therefore, hence having commuted the limits, it will be

$$Q = +\int \frac{y^{k-m}}{1+y^k} \frac{dy}{y}$$

from the limit $y = 0$ to $y = 1$. Now since, having calculated this integral, the letter y goes out of the calculation, we can write x instead of y so that

$$Q = \int \frac{x^{k-m}}{1+x^k} \cdot \frac{dx}{x},$$

having done which we will have

$$P + Q = \int \frac{x^m + x^{k-m}}{1+x^k} \cdot \frac{dx}{x}$$

from the limit $x = 0$ to $x = 1$. But not so long ago I demonstrated that the value of this integral extended from $x = 0$ to $x = 1$ is $= \frac{\pi}{k \sin \frac{m\pi}{k}}$. Therefore, hence the following not less remarkable theorem results.

Theorem

15. *The value of this integral formula*

$$\int \frac{x^m + x^{k-m}}{1+x^k} \cdot \frac{dx}{x}$$

extended from $x = 0$ to $x = 1$ is equal to the value of this integral

$$\int \frac{x^m}{1+x^k} \cdot \frac{dx}{x}$$

extended from the $x = 0$ to ∞.

16. Having discussed these things let us now go over to the integral formula propounded in the title, and to reduce it to the form treated up to now, recall the following reduction

$$\int \frac{x^{m-1}dx}{(1+x^k)^{\lambda+1}} = \frac{Ax^m}{(1+x^k)^{\lambda}} + B\int \frac{x^{m-1}dx}{(1+x^k)^{\lambda}},$$

whence after differentiation the following equation results

$$\frac{x^{m-1}dx}{(1+x^k)^{\lambda+1}} = \frac{mAx^{m-1}dx}{(1+x^k)^{\lambda}} - \frac{\lambda kAx^{m+k-1}dx}{(1+x^k)^{\lambda+1}} + \frac{Bx^{m-1}dx}{(1+x^k)^{\lambda}},$$

which equation, divided by $x^{m-1}dx$ and multiplied $(1+x^k)^{\lambda}$, by bringing the negative term from the left-hand side to the right-hand side will be

$$\frac{1+\lambda kAx^k}{1+x^k} = mA + B,$$

which equation can manifestly only hold, if $\lambda kA = 1$ or $A = \frac{1}{\lambda k}$, whence it will be $1 = mA + B = \frac{m}{\lambda k} + B$, and so it will be $B = 1 - \frac{m}{\lambda k}$.

17. Having found these values for the letters A and B we first assumed that the integrals are taken in such a way that they vanish for $x = 0$; then having put $x = \infty$, since the exponent n is supposed to be smaller than k, the absolute term containing the letter A vanishes, so that in this case $x = \infty$

$$\int \frac{x^{m-1}dx}{(1+x^k)^{\lambda+1}} = \left(1 - \frac{m}{\lambda k}\right)\int \frac{x^{m-1}dx}{(1+x^k)^{\lambda}}.$$

If we now take $\lambda = 1$ first, since before we found that for the same case $x = \infty$

$$\int \frac{x^{m-1}dx}{1+x^k} = \frac{\pi}{k\sin\frac{m\pi}{k}},$$

we will have the value of this integral

$$\int \frac{x^{m-1}dx}{(1+x^k)^2} = \left(1 - \frac{m}{k}\right)\frac{\pi}{k\sin\frac{m\pi}{k}},$$

if the integral is also extended from $x = 0$ to $x = \infty$.

18. If we now in like manner put $\lambda = 2$, for the same limits of integration one will find

$$\int \frac{x^{m-1}dx}{(1+x^k)^3} = \left(1 - \frac{m}{k}\right)\left(1 - \frac{m}{2k}\right)\frac{\pi}{k\sin\frac{m\pi}{k}};$$

if in the same way continuously larger values are attributed to the letter λ, one will find the following remarkable forms of the integrals

$$\int \frac{x^{m-1}dx}{(1+x^k)^4} = \left(1 - \frac{m}{k}\right)\left(1 - \frac{m}{2k}\right)\left(1 - \frac{m}{3k}\right)\frac{\pi}{k\sin\frac{m\pi}{k}},$$
$$\int \frac{x^{m-1}dx}{(1+x^k)^5} = \left(1 - \frac{m}{k}\right)\left(1 - \frac{m}{2k}\right)\left(1 - \frac{m}{3k}\right)\left(1 - \frac{m}{4k}\right)\frac{\pi}{k\sin\frac{m\pi}{k}},$$
$$\int \frac{x^{m-1}dx}{(1+x^k)^6} = \left(1 - \frac{m}{k}\right)\left(1 - \frac{m}{2k}\right)\left(1 - \frac{m}{3k}\right)\left(1 - \frac{m}{4k}\right)\left(1 - \frac{m}{5k}\right)\frac{\pi}{k\sin\frac{m\pi}{k}},$$

etc.

19. Hence, if the letter n denotes an arbitrary integer for the formula in the title, if its integral is extended from $x = 0$ to $x = \infty$, its value reads as follows:

$$\left(1 - \frac{m}{k}\right)\left(1 - \frac{m}{2k}\right)\left(1 - \frac{m}{3k}\right)\left(1 - \frac{m}{4k}\right)\cdots\left(1 - \frac{m}{(n-1)k}\right)\frac{\pi}{k\sin\frac{m\pi}{k}},$$

which is therefore equal to this integral formula

$$\int \frac{x^{m-1}dx}{(1+x^k)^n}.$$

20. Here it is certainly only possible to take integer numbers for n; but by the method of interpolation on the other hand, which was explained at various places in more detail, it is possible to extend this integration also to cases, in which the exponent n is a fractional number. For, if the following integral formulas are extended from $y = 0$ to $y = 1$, in general the value of our propounded formula can be represented this way

$$\int \frac{x^{m-1}dx}{(1+x^k)^n} = \frac{\pi}{k\sin\frac{m\pi}{k}} \cdot \frac{\int y^{nk-m-1}dy(1-y^k)^{\frac{m}{k}-1}}{\int y^{k-m-1}dy(1-y^k)^{\frac{m}{k}-1}}.$$

Hence, if $m = 1$ and $k = 2$, it follows that

$$\int \frac{dx}{(1+xx)^n} = \frac{\pi}{2}\int \frac{y^{2(n-1)}dy}{\sqrt{1-yy}} : \int \frac{dy}{\sqrt{1-yy}} = \int \frac{y^{2(n-1)}dy}{\sqrt{1-yy}}.$$

So, if $n = \frac{3}{2}$, it will be

$$\int \frac{dx}{(1+xx)^{\frac{3}{2}}} = \int \frac{ydy}{\sqrt{1-yy}},$$

whose validity is immediately clear, since the first indefinite integral is $\frac{x}{\sqrt{1+xx}}$, the second $= 1 - \sqrt{1-yy}$, which for $x = \infty$ and $y = 1$ are obviously equal. Furthermore, for this general integration it will be helpful to have noted that the exponent can not be taken smaller than 1, since otherwise the values of both integrals would become infinite.